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Explicit construction of the spin-4 Casimir operator in the coset model $\hat{SO}(5)_1 \times \hat{SO}(5)_m/\hat{SO}(5)_{1+m}$

Changhyun Ahn†‡§
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, NY 11794-3840, USA

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Abstract. We generalize the coset constructions to the dimension-5/2 operator for $\hat{so}(5)$ and compute the fourth-order Casimir invariant in the coset model $\hat{SO}(5)_1 \times \hat{SO}(5)_m/\hat{SO}(5)_{1+m}$ with the generic unitary minimal $c < 5/2$ series that can be viewed as perturbations of the $m \to \infty$ limit, which has previously been investigated in the $c = 5/2$ realization of the free fermion model.

Extensions of conformal symmetry have played an important role in the systematic study of two-dimensional conformal quantum field theories. It is of great consequence to realize that in a given model of conformal field theory the true symmetry algebra is bigger than the conformal algebra alone. The representations of such a model can be enhanced by studying the extra symmetry. Very recently, nonlinear higher-spin extensions of the Virasoro algebra, so called W-algebras, have been reviewed in [1].

After two scalar free-field realizations [2] of Zamolodchikov's W_3 algebra [3], Fateev and Lukyanov [4] have extended it to the infinite-dimensional associative algebras WA_n, WD_n and WB_n, based on the finite Lie algebras A_n, D_n, and B_n, respectively. In particular, the spins of the fields of the WB_n algebra are not related to the exponents of $B_n = so(2n+1)$, but instead to those of the Lie superalgebra $B(0, n) = osp(1, 2n)$; $1, 3, \ldots, 2n-1$ and $n - 1/2$ (the exponents plus 1) [5]. For $n = 1$, the WB_1 algebra corresponds to the $N = 1$ superconformal algebra.

Extended symmetries are easy to deal with in conformal field theories based on the coset constructions [6, 7]. In this approach, the unitary representations of the Casimir algebra ($c < l$, where l is the rank of the algebra) can be obtained from the cosets for the algebras ADE. For the non-simply laced algebra, B_n, this was further discussed in [5]. The discrete series of c values [4] from free-field construction coincide with values obtained from coset models associated with the $\hat{B}_n = \hat{so}(2n+1)$ algebra.

The existence of the WB_2 algebra, which is associative for all values of c, has been shown in [8] by explicitly using the perturbative conformal bootstrap. We were able to reproduce their findings by exploiting the graded Jacobi identity for the Laurent expansion modes of generating currents and illustrated a realization of the $c = 5/2$ free fermion model from the basic fermion fields and finally confirmed that the *bosonic* currents in the WB_2 algebra are the Casimirs of $\hat{so}(5)$ [9].

† E-mail address: ahn@max.physics.sunysb.edu
‡ Researcher, Natural Science Institute, Yonsei University, Seoul, 120-749, Korea.
§ Present address: Bogoliubov Theoretical Laboratory, JINR, Dubna, Head Post Office, PO Box 79, 101000, Moscow, Russia.

In this paper, we want to generalize the algebraic structure of [9] and construct WB_2 currents in the coset model of the generic unitary minimal discrete $c < 5/2$ series and explain briefly how to count the $so(5)$ singlets in the $c = 5/2$ free fermion model by analysing a generating function.

We take a look at the coset model $\hat{SO}(5)_1 \times \hat{SO}(5)_m/\hat{SO}(5)_{1+m}$ which can be regarded as perturbations of the $m \to \infty$ model that was considered in [9]. Denoting the generators of the algebra $g = \hat{so}(5) \oplus \hat{so}(5)$ by $E^{ab}_{(1)}(z)$ and $E^{ab}_{(2)}(z)$, of level 1 and m, respectively, and those of the diagonal subalgebra $g' = \hat{so}(5)$, which has level $m' = 1 + m$, as $E'^{ab}(z)$, we have the relation

$$E'^{ab}(z) = E^{ab}_{(1)}(z) + E^{ab}_{(2)}(z). \tag{1}$$

The indices a and b take values in the adjoint representation of $so(5)$ and $a, b = 1, 2, \ldots, 5$. The $\hat{so}(5)$ algebra has 10 fields ($E^{ab}(z) = -E^{ba}(z)$). The coset Virasoro generator $\tilde{T}(z)$ is, as usual, given by

$$\begin{aligned}\tilde{T}(z) &= T_{(1)}(z) + T_{(2)}(z) - T'(z)\\ &= -\frac{1}{16}E^{ab}_{(1)}E^{ab}_{(1)}(z) - \frac{1}{4(m+3)}E^{ab}_{(2)}E^{ab}_{(2)}(z) + \frac{1}{4(m+4)}E'^{ab}E'^{ab}(z)\end{aligned} \tag{2}$$

which commutes with $E'^{ab}(z)$. The coset central charge of the unitary minimal models for WB_2 is

$$\tilde{c} = c(WB_2) = \frac{5}{2} + \frac{10m}{m+3} - \frac{10(m+1)}{m+4} = \frac{5}{2}\left[1 - \frac{12}{(m+3)(m+4)}\right] \tag{3}$$

where $m = 1, 2, \ldots$.

Our next step is to extend the GKO coset construction to the dimension 5/2 operator associated with $\hat{so}(5)$; we follow the analysis in [10]. In order to write down the coset analogue $\tilde{U}(z)$ of dimension 5/2, we allow a general linear combination of the terms

$$\epsilon^{abcde}\psi^a_{(1)}E^{bc}_{(1)}E^{de}_{(1)} \qquad \epsilon^{abcde}\psi^a_{(1)}E^{bc}_{(1)}E^{de}_{(2)} \qquad \epsilon^{abcde}\psi^a_{(1)}E^{bc}_{(2)}E^{de}_{(2)} \tag{4}$$

and proceed by imposing that $\tilde{U}(z)$ transforms under $\tilde{T}(z)$ as a dimension 5/2 primary field. We have found that this uniquely fixes $\tilde{U}(z)$ up to a normalization factor $A(1, m)$

$$\begin{aligned}\tilde{U}(z) = A(1, m)\epsilon^{abcde}\Bigg[&\psi^a_{(1)}E^{bc}_{(1)}E^{de}_{(1)}\\ &- \frac{10}{m}\psi^a_{(1)}E^{bc}_{(1)}E^{de}_{(2)} + \frac{15}{m(m+2)}\psi^a_{(1)}E^{bc}_{(2)}E^{de}_{(2)}\Bigg](z)\end{aligned} \tag{5}$$

which are singlets under the underlying $so(5)$ subalgebra of $g' = \hat{so}(5)$. One can show that $\tilde{U}(z)$ has an element of the coset zero contraction with $E'^{ab}(z)$. The normalization factor $A(1, m)$ can be fixed as

$$A(1, m) = \frac{m}{120}\sqrt{\frac{(m+2)}{(m+3)(m+4)(m+5)}} \tag{6}$$

by investigating the $1/(z-w)^5$ term of $\tilde{U}(z)\tilde{U}(w)$. The expression for $\tilde{U}(z)$ has already been proposed in [5] with different normalizations. One can easily see that $\tilde{U}(z)$ reduces to $U(z) = \frac{1}{120}\epsilon^{abcde}\psi^a_{(1)}E^{bc}_{(1)}E^{de}_{(1)}$ [9] as $m \to \infty$. After a tedious calculation, repeatedly using Wick's theorem for the operator product expansion [10], one finds

$$
\begin{aligned}
\tilde{U}(z)\tilde{U}(w) = & \frac{1}{(z-w)^5}\frac{m(m+7)}{(m+3)(m+4)} \\
& + \frac{1}{(z-w)^3}\frac{1}{(m+4)}\left[-\frac{m}{8}E^{ab}_{(1)}E^{ab}_{(1)} + E^{ab}_{(1)}E^{ab}_{(2)} - \frac{1}{2(m+3)}E^{ab}_{(2)}E^{ab}_{(2)}\right](w) \\
& + \frac{1}{(z-w)^2}\frac{1}{(m+4)}\left[-\frac{m}{8}E^{ab}_{(1)}\partial E^{ab}_{(1)} + \frac{1}{2}\partial(E^{ab}_{(1)}E^{ab}_{(2)})\right. \\
& \left. - \frac{1}{2(m+3)}E^{ab}_{(2)}\partial E^{ab}_{(2)}\right](w) + \frac{1}{(z-w)}\frac{m^2(m+2)}{14400(m+3)(m+4)(m+5)} \\
& \times\left[\frac{7200(m+2)}{m}\psi^a_{(1)}\partial\psi^a_{(1)}\psi^b_{(1)}\partial\psi^b_{(1)} - \frac{2400(m-1)(m+3)}{m(m+2)}\psi^a_{(1)}\partial^3\psi^a_{(1)}\right. \\
& - \frac{100}{m}\epsilon^{abcde}\epsilon^{afghi}(E^{bc}_{(1)}E^{de}_{(1)})(E^{fg}_{(1)}E^{hi}_{(2)}) \\
& - \frac{400(m+7)}{m^2}\epsilon^{abcde}\epsilon^{abfgh}\psi^f_{(1)}\partial(\psi^c_{(1)}E^{de}_{(1)})E^{gh}_{(2)} \\
& - \frac{3600(m^2+5m+12)}{m^2(m+2)}\partial^2E^{ab}_{(1)}E^{ab}_{(2)} + \frac{3600(m+3)}{m^2}E^{ab}_{(1)}\partial^2E^{ab}_{(2)} \\
& + \frac{50(5m+16)}{m^2(m+2)}\epsilon^{abcde}\epsilon^{afghi}(E^{bc}_{(1)}E^{de}_{(1)})(E^{fg}_{(2)}E^{hi}_{(2)}) \\
& + \frac{300}{m^2}\epsilon^{abcde}\epsilon^{afghi}(E^{bc}_{(1)}E^{de}_{(2)})(E^{fg}_{(1)}E^{hi}_{(2)}) \\
& + \frac{600(m+5)}{m^2(m+2)}\epsilon^{abcde}\epsilon^{abfgh}\psi^f_{(1)}(\partial(\psi^c_{(1)}E^{de}_{(2)})E^{gh}_{(2)}) \\
& - \frac{900(m+3)}{m^2(m+2)^2}\epsilon^{abcde}\epsilon^{afghi}(E^{bc}_{(1)}E^{de}_{(2)})(E^{fg}_{(2)}E^{hi}_{(2)}) \\
& \left. + \frac{225}{m^2(m+2)^2}\epsilon^{abcde}\epsilon^{afghi}(E^{bc}_{(2)}E^{de}_{(2)})(E^{fg}_{(2)}E^{hi}_{(2)})\right](w) + \cdots. \qquad (7)
\end{aligned}
$$

The parentheses denote the 'normal ordered product' between operators at coincident points. It can be defined as a contour integral for the operators A and B

$$(AB)(z) = \frac{1}{2\pi i}\oint_z \frac{dw}{w-z}A(w)B(z). \qquad (8)$$

How can we extract the dimension four coset field $\tilde{V}(z)$ from the singular part of $1/(z-w)$ in the above operator product expansion? The convenient way to do it is to make all the composite operators 'fully normal ordered products' [10]. Then, using the rearrangement lemmas [10] extensively, we arrive at the following results

$$\tilde{U}(z)\tilde{U}(w) = \frac{1}{(z-w)^5}\frac{2}{5}\tilde{c} + \frac{1}{(z-w)^3}2\tilde{T}(w) + \frac{1}{(z-w)^2}\partial\tilde{T}(w) + \frac{1}{(z-w)}\left[\frac{3}{10}\partial^2\tilde{T}(w) + \frac{27}{(5\tilde{c}+22)}\tilde{\Lambda}(w) + \sqrt{\frac{6(14\tilde{c}+13)}{(5\tilde{c}+22)}}\tilde{V}(w)\right] + \cdots \tag{9}$$

and

$$\begin{aligned}\tilde{V}(z) = {} & \frac{1}{4(m+4)(m+5)\sqrt{3(2m+1)(2m+13)(23m^2+161m+176)}} \\ & \times [a\partial E^{ab}_{(1)}\partial E^{ab}_{(1)} + bE^{ab}_{(1)}\partial^2 E^{ab}_{(1)} + tE^{ab}_{(1)}E^{ab}_{(1)}E^{cd}_{(1)}E^{cd}_{(1)} \\ & + d\partial E^{ab}_{(2)}\partial E^{ab}_{(2)} + eE^{ab}_{(2)}\partial^2 E^{ab}_{(2)} + f\partial^2 E^{ab}_{(1)}E^{ab}_{(2)} \\ & + g\partial E^{ab}_{(1)}\partial E^{ab}_{(2)} + hE^{ab}_{(1)}\partial^2 E^{ab}_{(2)} + iE^{ab}_{(1)}E^{ab}_{(1)}E^{cd}_{(1)}E^{cd}_{(2)} \\ & + jE^{ab}_{(1)}\partial E^{ac}_{(1)}E^{bc}_{(2)} + kE^{ab}_{(1)}E^{ab}_{(1)}E^{cd}_{(2)}E^{cd}_{(2)} + lE^{ab}_{(1)}E^{cd}_{(1)}E^{ab}_{(2)}E^{cd}_{(2)} \\ & + sE^{ab}_{(1)}E^{ac}_{(2)}\partial E^{bc}_{(2)} + nE^{ab}_{(1)}E^{ab}_{(2)}E^{cd}_{(2)}E^{cd}_{(2)} + oE^{ab}_{(2)}E^{ab}_{(2)}E^{cd}_{(2)}E^{cd}_{(2)} \\ & + pE^{ab}_{(1)}E^{ac}_{(1)}E^{cd}_{(2)}E^{bd}_{(2)} + qE^{ab}_{(1)}E^{cd}_{(2)}E^{ac}_{(2)}E^{bd}_{(2)} + rE^{ab}_{(2)}E^{cd}_{(2)}E^{ac}_{(2)}E^{bd}_{(2)}](z)\end{aligned} \tag{10}$$

where

$$\begin{aligned}
& a = \tfrac{3}{4}m(m+2)(2m+1)(3m+17) \qquad b = -\tfrac{1}{2}m(m+2)(2m+1)(3m+17) \\
& t = -\tfrac{1}{16}m(m+2)(2m+1)(2m+11) \\
& d = \frac{3(2m^4+28m^3+33m^2-455m-688)}{8(m+2)(m+3)} \\
& e = -\frac{(2m^4+28m^3+33m^2-455m-688)}{4(m+2)(m+3)} \\
& f = \tfrac{1}{4}(m+2)(2m+1)(45m+259) \qquad g = -\tfrac{3}{2}(m^3+58m^2+349m+312) \\
& h = \frac{(2m^4+43m^2-93m^2-2084m-2836)}{4(m+2)} \\
& i = (m+2)(2m+1)(2m+11) \qquad j = (m+2)(2m+1)(7m+41) \\
& k = \tfrac{3}{8}(10m^2+73m+88) \qquad l = \tfrac{1}{2}(2m+1)(7m+41) \\
& n = -\frac{(7m^2+49m+43)}{(m+2)} \qquad o = \frac{(7m^2+49m+43)}{4(m+2)(m+3)} \\
& p = -(23m^2+161m+176) \qquad q = \frac{(23m^2+161m+176)}{(m+2)} \\
& r = -\frac{(23m^2+161m+176)}{4(m+2)(m+3)} \qquad s = \frac{(5m^3-42m^2-543m-716)}{(m+2)}.
\end{aligned} \tag{11}$$

The result for $V(z)$ in [9] is recovered in the limit $m \to \infty$

$$\tilde{V}(z) \to \frac{1}{8\sqrt{69}}\left[\frac{9}{2}\partial E^{ab}_{(1)}\partial E^{ab}_{(1)} - 3E^{ab}_{(1)}\partial^2 E^{ab}_{(1)} - \frac{1}{4}E^{ab}_{(1)}E^{ab}_{(1)}E^{cd}_{(1)}E^{cd}_{(1)}\right](z)$$
$$= -\frac{1}{40\sqrt{69}}E^{ab}_{(1)}E^{cd}_{(1)}E^{ac}_{(1)}E^{bd}_{(1)}(z) = V(z). \tag{12}$$

Comparing with the results of [9], we see that all the operators are replaced by their coset analogues and the central charge is given by (3). The one thing which we would like to stress is the fact that the nine independent fields containing the derivatives in the above expression for $\tilde{V}(z)$ can be reexpressed in terms of the products of four E^{ab}'s. As an example

$$E^{ab}_{(1)}E^{ac}_{(2)}\partial E^{bc}_{(2)}(z) = -\tfrac{1}{3}[E^{ab}_{(1)}E^{ac}_{(2)}E^{bd}_{(2)}E^{cd}_{(2)} - E^{ab}_{(1)}E^{cd}_{(2)}E^{ac}_{(2)}E^{bd}_{(2)}](z)$$
$$- \tfrac{1}{4}[E^{ab}_{(1)}E^{cd}_{(2)}E^{ab}_{(2)}E^{cd}_{(2)} - E^{ab}_{(1)}E^{ab}_{(2)}E^{cd}_{(2)}E^{cd}_{(2)}](z). \tag{13}$$

Finally, $\tilde{V}(z)$ can be written as

$$[C_1(m)\delta^{ea}\delta^{fb}\delta^{gc}\delta^{hd} + C_2(m)\delta^{ea}\delta^{fc}\delta^{gb}\delta^{hd}]E^{ab}E^{cd}E^{ef}E^{gh}(z)$$
$$+ [C_3(m)\delta^{ea}\delta^{fb}\delta^{gc}\delta^{hd} + C_4(m)\delta^{ea}\delta^{fc}\delta^{gb}\delta^{hd}]E^{ab}E^{ef}E^{cd}E^{gh}(z)$$
$$+ C_5(m)\delta^{ea}\delta^{fc}\delta^{gb}\delta^{hd}E^{ab}E^{ef}E^{gh}E^{cd}(z) \tag{14}$$

where $E^{ab}(z)$ is $E^{ab}_{(1)}(z)$ or $E^{ab}_{(2)}(z)$, the C's are some functions of m and δ^{ab} is an invariant tensor of $so(5)$. Therefore $\tilde{V}(z)$ is really the fourth-order Casimir operator for $\hat{so}(5)$. On the other hand, the field contents of $\partial^2\tilde{T}(z)$ and $\tilde{\Lambda}(z) = \tilde{T}^2(z) - \frac{3}{10}\partial^2\tilde{T}(z)$ are the same as those of $\tilde{V}(z)$ except that they do *not* have the terms

$$E^{ab}_{(1)}E^{ac}_{(1)}E^{cd}_{(2)}E^{bd}_{(2)} \qquad E^{ab}_{(1)}E^{cd}_{(2)}E^{ac}_{(2)}E^{bd}_{(2)} \qquad E^{ab}_{(2)}E^{cd}_{(2)}E^{ac}_{(2)}E^{bd}_{(2)}. \tag{15}$$

In order to check that $\tilde{V}(z)$ is a dimension four primary field with respect to $\tilde{T}(z)$, we should compute, by explicit calculations, the operator product expansions of $\tilde{T}(z)$ with 18 fields (10). As a check on this, the fact that $\tilde{T}(z)$ commutes with $E'^{ab}(z)$ has been repeatedly used. We list the operator product expansions $\tilde{T}(z)$ with the fields of (15)

$$\tilde{T}(z)E^{ab}_{(1)}E^{ac}_{(1)}E^{cd}_{(2)}E^{bd}_{(2)}(w)$$
$$= -\frac{1}{(z-w)^6}\frac{180m}{(m+4)} + \frac{1}{(z-w)^4}\frac{1}{(m+4)}[-4mE^{ab}_{(2)}E^{ab}_{(2)}$$
$$- 4mE^{ab}_{(1)}E^{ab}_{(1)} + (19m+12)E^{ab}_{(1)}E^{ab}_{(2)}](w)$$
$$+ \frac{1}{(z-w)^3}\frac{9}{2(m+4)}[-(m+3)E^{ab}_{(1)}\partial E^{ab}_{(2)} - 4\partial E^{ab}_{(1)}E^{ab}_{(2)} + E^{ab}_{(2)}\partial E^{ab}_{(2)}$$
$$+ mE^{ab}_{(1)}\partial E^{ab}_{(1)}](w) + O\left(\frac{1}{(z-w)^2}\right)$$

$$\tilde{T}(z)E^{ab}_{(1)}E^{cd}_{(2)}E^{ac}_{(2)}E^{bd}_{(2)}(w)$$

$$= \frac{1}{(z-w)^4}\frac{3}{(m+4)}[3(m-1)E^{ab}_{(2)}E^{ab}_{(2)} + (3m+2)E^{ab}_{(1)}E^{ab}_{(2)}](w) + \frac{1}{(z-w)^3}\frac{3(6m-1)}{2(m+4)}[\partial E^{ab}_{(1)}E^{ab}_{(2)} - E_{(1)}\partial E^{ab}_{(2)}] + \mathrm{O}\left(\frac{1}{(z-w)^2}\right) \tag{16}$$

$$\tilde{T}(z)E^{ab}_{(2)}E^{cd}_{(2)}E^{ac}_{(2)}E^{bd}_{(2)}(w)$$

$$= -\frac{1}{(z-w)^4}\frac{3}{(m+4)}[6(m-1)E^{ab}_{(2)}E^{ab}_{(2)} + (6m-1)E^{ab}_{(1)}E^{ab}_{(2)}](w) + \frac{1}{(z-w)^3}\frac{3(6m-1)}{(m+4)}[E^{ab}_{(1)}\partial E^{ab}_{(2)} - \partial E^{ab}_{(1)}E^{ab}_{(2)}](w) + \mathrm{O}\left(\frac{1}{(z-w)^2}\right).$$

The results obtained so far can be summarized as follows: We have found three currents of (2), (5) and (10) in the coset model (and in the $c = 5/2$ free fermion model). A complete evaluation of the operator product algebras $\tilde{U}(z)\tilde{V}(w)$ and $\tilde{V}(z)\tilde{V}(w)$ is straightforward in principle but not in practice. We will therefore turn to a different method.

The Neveu–Schwarz sector of the five fermion model decomposes into two irreducible highest weight modules of $\hat{so}(5)$ at level one. In this circumstance one obtains the following expression for the generating functional of states in the Neveu–Schwarz sector of the five fermion model that are singlets under $so(5)$ [5]

$$\chi^{\mathrm{NS}}_{\mathrm{singlets}}(q) = \frac{(1-q)(1-q)(1-q^2)(1-q^3)}{(1+q^{1/2})(1+q^{3/2})}\prod_{k=1}^{\infty}\frac{(1+q^{k-1/2})}{(1-q^k)^2}. \tag{17}$$

The factor $(1-q)(1-q)(1-q^2)(1-q^3)$ in the numerator corresponds to the fact that the modes of L_{-1}, V_{-1}, V_{-2} and V_{-3} annihilate the vacuum by requiring the regularity of the vacuum. In other words, they generate singular vectors. By counting the singlets in the factor module, the vacuum irreducible Verma module, we can recognize the generating currents that are in the chiral algebra [5, 11]. Therefore we can easily read off the conformal dimensions

$$\chi^{\mathrm{NS}}_{\mathrm{singlets}}(q) = \frac{\phi_{5/2}(q)}{\phi_2(q)\phi_4(q)} = 1 + q^2 + q^{5/2} + q^3 + q^{7/2} + 3q^4 + 2q^{9/2} + 3q^5 + 3q^{11/2} + 7q^6 + 6q^{13/2} + 8q^7 + \mathrm{O}(q^{15/2}) \tag{18}$$

where we have defined the 'modified Euler function'

$$\phi_\Delta(q) = \begin{cases} \prod_{k=\Delta}^{\infty}(1-q^k) & \text{if } \Delta \text{ is integer} \\ \prod_{k=\Delta}^{\infty}(1+q^{k-1}) & \text{if } \Delta \text{ is half integer.} \end{cases} \tag{19}$$

All the states in the vacuum Verma module obtained by acting with the creation modes of the currents in the associated Casimir algebra are invariant under the finite horizontal subalgebra. Of course, this is backed up by the closed operator algebras [9]. Equation (17) gives the number of independent states at each level in the Fock space of two boson fields and a fermionic field. A basis for the seven-dimensional eigenspace of $so(5)$ singlets with L_0 eigenvalue six is

$$\begin{array}{lll} L_{-6}|0\rangle_{\rm NS} & V_{-6}|0\rangle_{\rm NS} & L_{-2}L_{-2}L_{-2}|0\rangle_{\rm NS} \\ L_{-4}L_{-2}|0\rangle_{\rm NS} & V_{-4}L_{-2}|0\rangle_{\rm NS} & L_{-3}L_{-3}|0\rangle_{\rm NS} \\ U_{-\frac{7}{2}}U_{-\frac{5}{2}}|0\rangle_{\rm NS}. & & \end{array} \tag{20}$$

The explicit check of the absence of a null state of dimension six is straightforward.

From the above argument of the character for an irreducible representation, the structure of a Casimir algebra, the WB_2 algebra, is generated by three currents, which have dimension $2, 4, 5/2$, respectively. The operator algebras of these currents are expected to be similar for the five free fermion model *and* for the generic coset model. The coset models, at least for large m, can be viewed as perturbations of the limit model at $c = 5/2$ and will share its algebraic structure. This situation is similar to the case in [11]. It would be interesting to relate the operator content of (2), (5) and (10) to that of the free-field construction [4].

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References

[1] Bouwknegt P and Schoutens K 1993 W-symmetry in conformal field theory *Phys. Rep.* **223** 183

[2] Fateev V A and Zamolodchikov A B 1987 Conformal field theory models in two dimensions having Z_3 symmetry *Nucl. Phys.* B **280** 644

[3] Zamolodchikov A B 1986 Infinite additional symmetries in two-dimensional conformal quantum field theory *Theor. Math. Phys.* **65** 1205

[4] Lukyanov S L and Fateev V A 1988 Additional symmetries and exactly soluble models in two-dimensional conformal field theory *Preprints* ITF-88-74R, ITF-88-75R and ITF-88-76R, Institute of Theoretical Physics, Ukranian Academy of Sciences, Kiev

[5] Watts G M T 1990 WB algebra representation theory *Nucl. Phys.* B **339** 177; 1990 W-algebras and coset models *Phys. Lett.* **245B** 65

[6] Bardakci K and Halpern M B 1971 New dual quark models *Phys. Rev.* D **3** 2493
Halpern M B 1971 The two faces of a dual pion–quark model *Phys. Rev.* D **4** 2398

[7] Goddard P, Kent A and Olive D 1985 Virasoro algebras and coset space models *Phys. Lett.* **152B** 88; 1986 Unitary representations of the Virasoro and super-Virasoro algebra *Commun. Math. Phys.* **103** 105
Friedan D, Qiu Z and Shenker S 1984 Conformal invariance, unitarity and critical exponents in two dimensions *Phys. Rev. Lett.* **52** 1575

[8] Figueroa-O'Farrill J M, Schrans S and Thielmans K 1991 On the Casimir algebra of B_2 *Phys. Lett.* **263B** 378

[9] Ahn C 1992 $c = 5/2$ free fermion model of WB_2 algebra *Int. J. Mod. Phys.* A **7** 6799

[10] Bais F A, Bouwknegt P, Schoutens K and Surridge M 1988 Extensions of the Virasoro algebra constructed from Kac–Moody algebras using higher-order Casimir invariants *Nucl. Phys.* B **304** 348; 1988 Coset construction for extended Virasoro algebras *Nucl. Phys.* B **304** 371

[11] Bouwknegt P 1988 Infinite-dimensional Lie algebras and Lie groups *Proc. CIRM-Luminy Conf.* ed V Kac (Singapore: World Scientific)
Ahn C, Schoutens K and Sevrin A 1991 The full structure of the super W_3 algebra *Int. J. Mod. Phys.* A **6** 3467